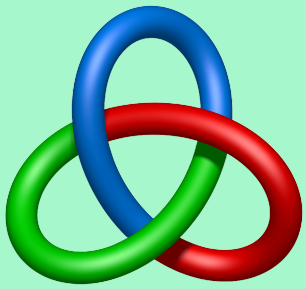
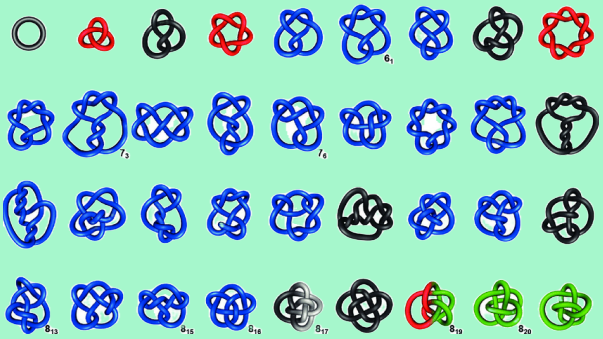


Intro to Knot Theory

Mentor: Peyton Wood



Prerequisites: None (Will not require MAT 108 or any prior knowledge)

Description: This seminar will introduce a group of students to the world of mathematical knots. We will mainly explore (1) what mathematical knots are, (2) how to tell knots apart from each other, (3) what uses knots have played and will continue to play in the real world.

Sources (no purchases necessary):

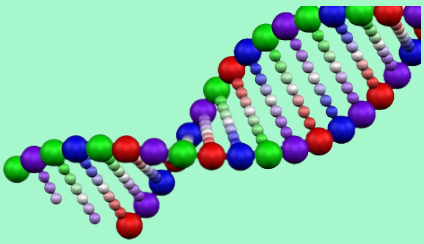
1. The Knot Book by Colin Adams
2. Quandles by Mohamed Elhamdadi and Sam Nelson

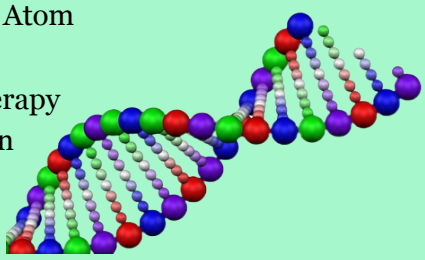
Course Outline:

Meetings will be a mixture of lecture style presentation, large group discussion, and working through examples in small groups. The goal is every meeting you will have the chance to learn something, ask questions, and get experience working on some practice problems with small groups.

After the first five weeks, participants will be asked to spend time researching and give their own short presentations on a knot theory topic we haven't covered yet. For the final week of the quarter, participants will vote on what topic they want to see covered.

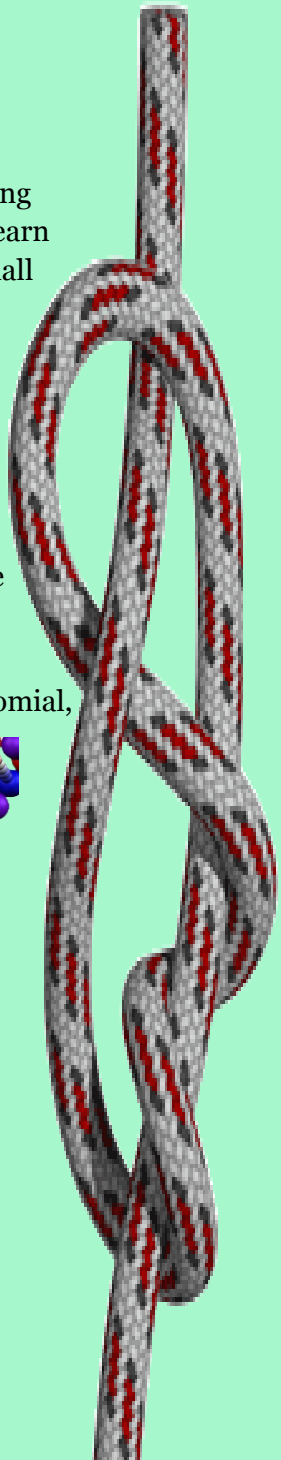
Week 1- No meeting. Via email we will find a meeting time for Weeks 2-10
Weeks 2-6 Topics- Define Knots, Links, Knot/Link Diagrams, Reidemeister Moves, The Unknot, Connected Sum of Knots, Prime and Composite Knots, Dowker Code, Oriented Knots, Reversal and Reflection of a Knot Diagram, Gauss Code, Knot Invariants, Tricolorability, Crossing Number, Unknotting Number, Bridge Number, Alexander Polynomial, Kauffman Bracket Skein Relation, Jones Polynomial





Weeks 7-9- Student Presentations
Suggested Options:

1. Knot History
2. Vortex Theory of the Atom
3. Knots and DNA
4. Knots and Chemotherapy
5. Knots and Encryption
6. Braids
7. Sticks
8. Conway Notation
9. Connection between knots and planar graphs

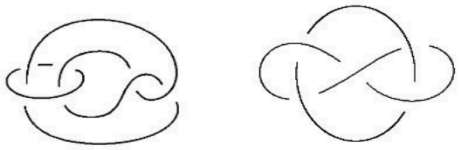


10. Torus Knots
11. Satellite Knots
12. Hyperbolic Knots
13. Alternating Knots
14. Chirality and Amphichirality
15. Knotted Molecules
16. Network Analysis using Knots

Week 10 (Optional)- Students will vote:
Quandles, Virtual Knots, Legendrian Knots, or Sliceness and Concordance

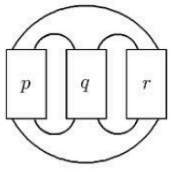
Some examples of problems we will work on:

Exercises. 1. Using Reidemeister moves, show that the diagrams below represent the same link. This link is known as the *Whitehead link*.

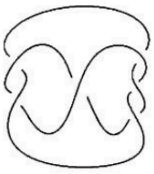


5. Use the Jones polynomial to prove that the right-handed and left-handed trefoils below are not equivalent.

3. Let p, q, r be three integers. A (p, q, r)-*pretzel link* is a knot or link of the form



where the boxes are replaced with stacks of p, q and r oriented crossings respectively (a negative value means use negative crossings – also note that the orientation of the crossings in the boxes may not extend to the whole link!) For example, the $(2, 1, -3)$ pretzel link is



How many components are possible in a pretzel link? What conditions on p, q and r ensure that we have a knot? A 2-component link?

What is the connected sum of any knot with the unknot?
Which of the first 5 knots are equivalent to their reflection? Reversal?
Is the connected sum of a tricolorable knot with any knot always tricolorable?